

# Chomsky Normal Form

Lecture 18  
Section 6.2

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# Outline

- 1 Chomsky Normal Form
  - Eliminate all mixed productions
  - Eliminate all long productions
- 2 Derivations in CNF
- 3 Assignment

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# Eliminate All Mixed Productions

## Definition (Mixed production)

A **mixed production** is a production whose right-hand side has length at least 2 and contains at least one terminal.

## Proof (Eliminate all mixed productions).

- For each terminal  $a$  appearing in a mixed production, add a production

$$A \rightarrow a$$

where  $A$  is a new variable.



# Example

## Example (Eliminate all mixed productions)

- Eliminate the mixed productions from the following grammar.

$$S \rightarrow SAS \mid \mathbf{bAa}$$

$$A \rightarrow \mathbf{aS} \mid \mathbf{Sb} \mid \mathbf{ab} \mid S \mid \lambda$$

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# Eliminate All Long Productions

## Definition (Mixed production)

A **long production** is a production whose right-hand side has length at least 3.



# Eliminate All Long Productions

Proof (Eliminate all long productions).

- Replace the long productions

$$A \rightarrow B_1 B_2 \dots B_k, (k \geq 3)$$

with

$$A \rightarrow B_1 C_1$$

$$C_1 \rightarrow B_2 C_2$$

$$C_2 \rightarrow B_3 C_3$$

⋮

$$C_{k-2} \rightarrow B_{k-2} C_{k-2}$$

$$C_{k-1} \rightarrow B_{k-1} B_k$$



# Example

## Example (Eliminate all long productions)

- Eliminate all long productions from the following grammar.

$$S \rightarrow SAS \mid CAB$$

$$A \rightarrow BS \mid SC \mid BC \mid \lambda$$

$$B \rightarrow \mathbf{a}$$

$$C \rightarrow \mathbf{b}$$

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# A Derivation in CNF

## Example (A CNF derivation)

- Use this grammar in CNF to derive the string **bababa**.

$$S \rightarrow SD \mid CE \mid SS \mid CB$$

$$A \rightarrow BC \mid SC \mid BS$$

$$B \rightarrow \mathbf{a}$$

$$C \rightarrow \mathbf{b}$$

$$D \rightarrow AS$$

$$E \rightarrow AB$$

# A Derivation in CNF

## Example (A CNF derivation)

$S \Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow$  **bababa**

# A Derivation in CNF

## Example (A CNF derivation)

$S \Rightarrow SS$   
 $\Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow$  **bababa**

# A Derivation in CNF

## Example (A CNF derivation)

$S \Rightarrow SS$   
 $\Rightarrow CBS$   
 $\Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow$  **bababa**

# A Derivation in CNF

## Example (A CNF derivation)

$S \Rightarrow SS$   
 $\Rightarrow CBS$   
 $\Rightarrow CBCE$   
 $\Rightarrow$   
 $\Rightarrow$   
 $\Rightarrow \mathbf{bababa}$



# A Derivation in CNF

## Example (A CNF derivation)

$S \Rightarrow SS$   
 $\Rightarrow CBS$   
 $\Rightarrow CBCE$   
 $\Rightarrow CBCAB$   
 $\Rightarrow$   
 $\Rightarrow \mathbf{bababa}$

# A Derivation in CNF

## Example (A CNF derivation)

$S \Rightarrow SS$   
 $\Rightarrow CBS$   
 $\Rightarrow CBCE$   
 $\Rightarrow CBCAB$   
 $\Rightarrow CBCBCB$   
 $\Rightarrow \mathbf{bababa}$

## Theorem

*If a grammar  $G$  is in CNF and a string  $w$  in  $L(G)$  has length  $n$ , then  $w$  is derived from  $G$  in exactly  $2n - 1$  steps.*

# Example

## Example (The Membership Problem for CFGs)

- Show that **bbbbaaa** is not in the language of the following grammar.

$$S \rightarrow SD \mid CE \mid SS \mid CB$$

$$A \rightarrow BC \mid SC \mid BS$$

$$B \rightarrow \mathbf{a}$$

$$C \rightarrow \mathbf{b}$$

$$D \rightarrow AS$$

$$E \rightarrow AB$$

# Example

## Example (The Membership Problem for CFGs)

- Put the grammar

$$E \rightarrow E + E \mid E * E \mid (E) \mid \mathbf{a} \mid \mathbf{b} \mid \mathbf{c}$$

into CNF.

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## Assignment

- Section 6.2 Exercises 1, 2, 3, 4, 5, 8, 9.